Towards Aggregating Lattice Signatures: Linear Aggregation With Compression

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Where and who am I?

Image: Open Street Map

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Lattice-based cryptography

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Lattice-based cryptography

& Module Learning With Errors (+ Variants)

Image: Open Street Map

Context & History

The purpose of cryptology:

- **confidentiality**
- **authenticity**
- **integrity**

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Digital Signatures ensure authenticity and integrity!

1976 First described by Diffie and Hellman [DH76]

1978 First realized by Rivest, Shamir and Adleman [RSA78]

1988 Rigorously defined security notions by Goldwasser, Micali and Rivest [GMR88]

Digital Signature Scheme $\Pi_S = (\mathsf{KGen}, \mathsf{Sig}, \mathsf{Vf})$

message 📄

$(\mathsf{sk}, \mathsf{vk}) \leftarrow \mathsf{KGen}$

$$📄, ✎ = \mathsf{Sig}(📄, \mathsf{sk}) \longrightarrow$$

$\{0, 1\} \leftarrow \mathsf{Vf}(\mathsf{vk}, 📄, ✎)$

Signature is **valid** if $1 \leftarrow \mathsf{Vf}$.
Correctness, unforgeability.

Digital Signature Scheme $\Pi_S = (\mathsf{KGen}, \mathsf{Sig}, \mathsf{Vf})$

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$$m_1, \sigma_1 = \mathsf{Sig}(m_1, \mathsf{sk}_1) \longrightarrow$$

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message m_2

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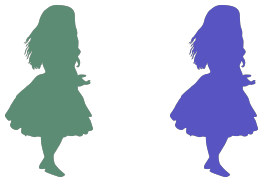
Q: Can we combine both (m_1, σ_1) and (m_2, σ_2) to something shorter?

And more generally for $N \gg 2$ many signatures?

Aggregate Signatures: AggSig and AggVf [BGLS03]



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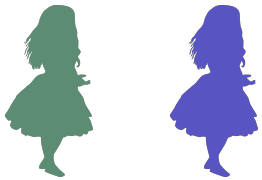

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Properties

Correctness

Public aggregation

Compactness

Unforgeability

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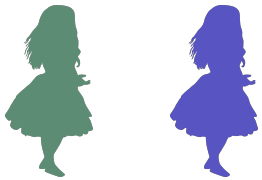

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only public input

m_1, m_2, σ →

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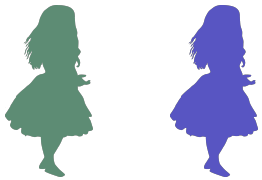

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in chosen-key model (later more)

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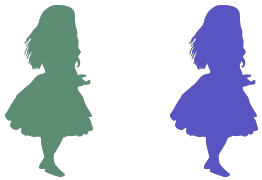

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Applications

Consensus Protocols

Certificate Chains

Blockchains

Research Question:

Can we construct a lattice-based

and compact aggregate signature scheme

with public aggregation ?

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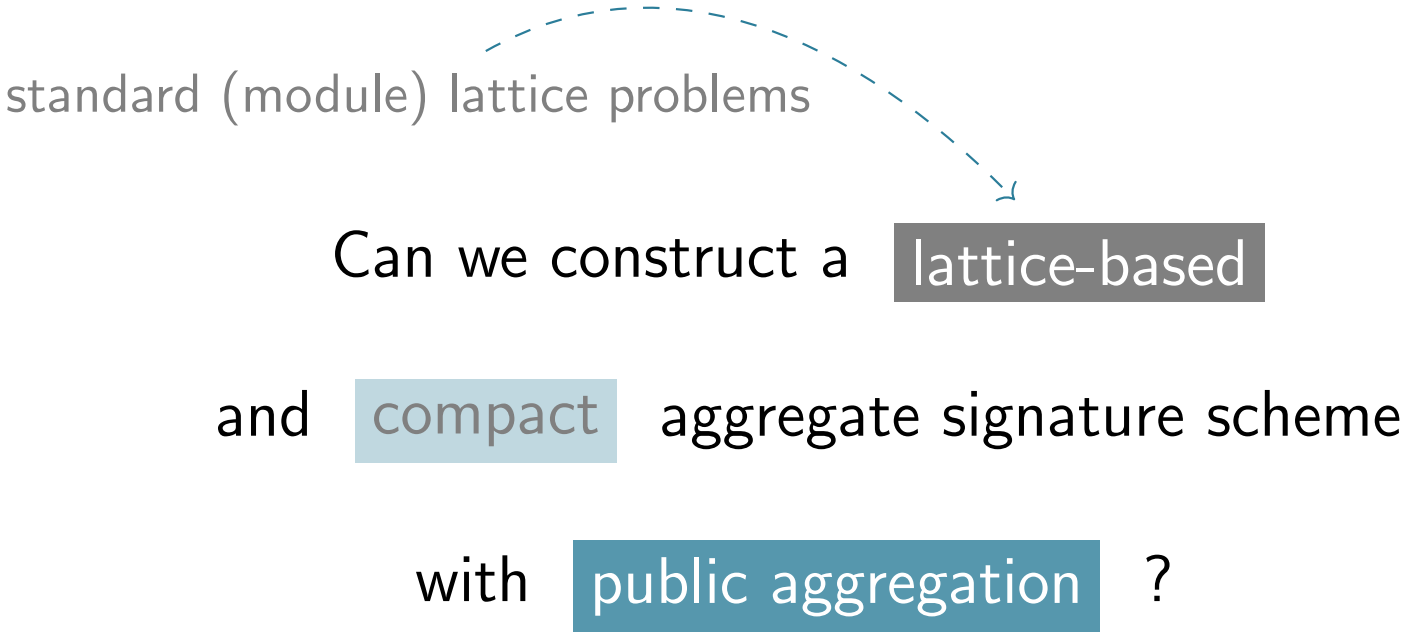

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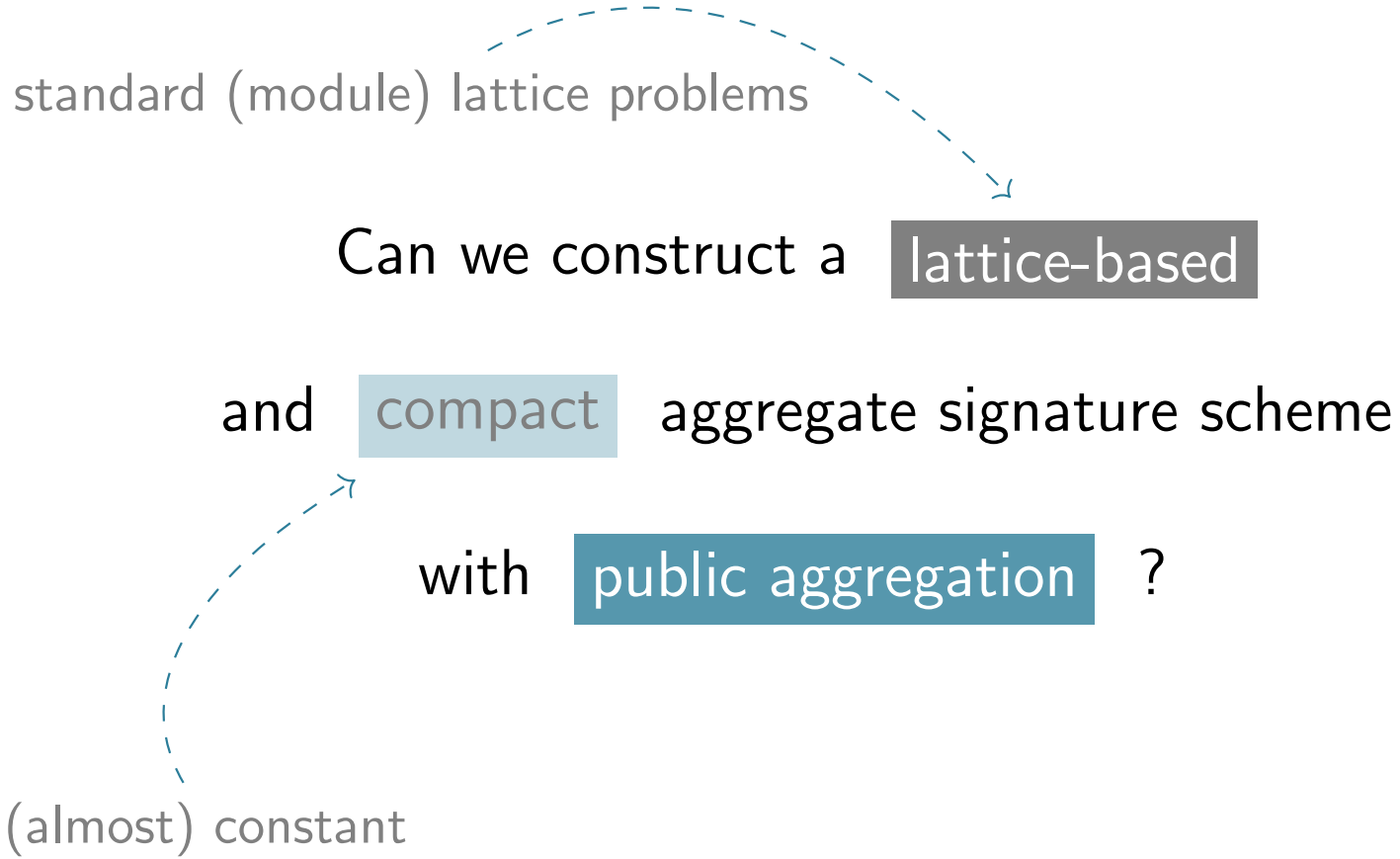

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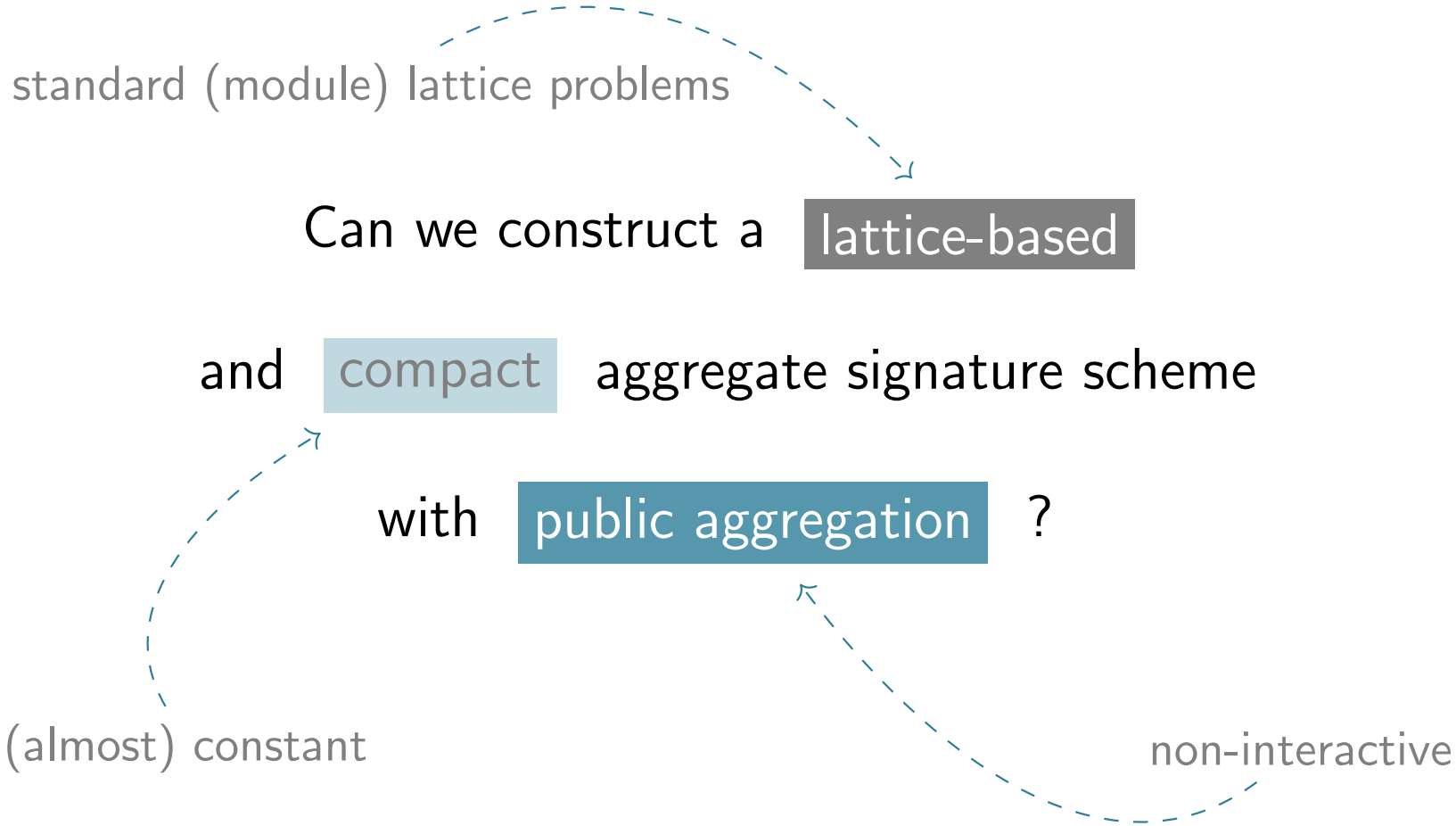

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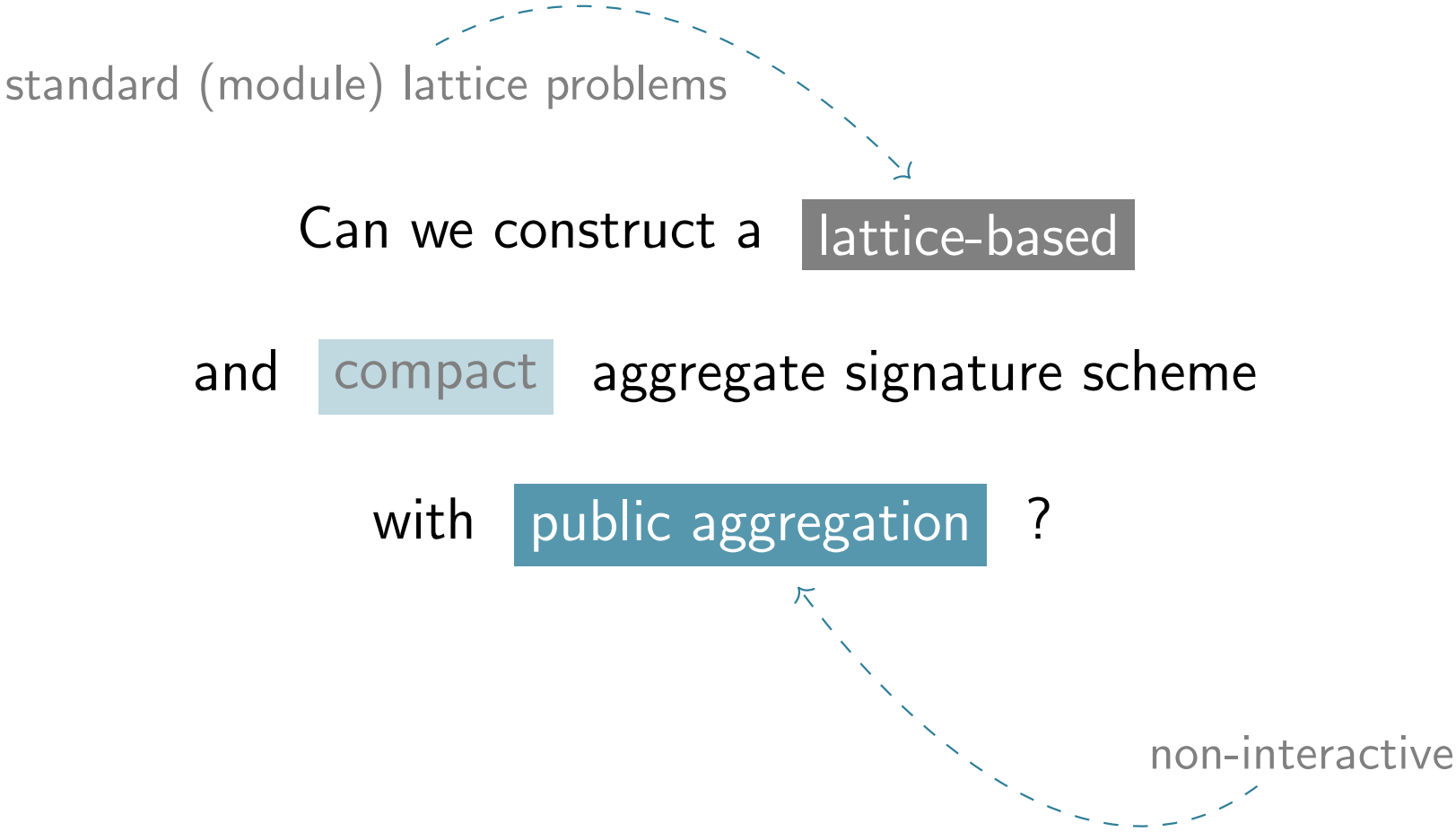

Today: concentrate on lattice-based and public aggregation, see

ia.cr/2021/263

Outline

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Fiat-Shamir with Aborts (FSwA) [Lyu09]

Let $R_q = \mathbb{Z}_q[x]/\langle x^n + 1\rangle$ and $A' \leftarrow U(R_q^{k\times\ell})$ defining $A = [A'|I_k]$ be public parameters and $H_c : \{0,1\}^* \to C$ be a random oracle

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n power of 2, q prime

k and ℓ small constants

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$y \leftarrow R^{k+\ell}$ small, $u = Ay$

$c = H_c(u, \text{message}) \in R$ small

$z = s \cdot c + y$ (rejection sampling)

$\xrightarrow{\text{message}, = (c, z)}$

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if $c = H_c(Az - tc, \text{msg})$

and z small, accept

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$$\begin{aligned} & Az - tc \\ = \; & A(sc + y) - (As)c \\ = \; & Ay \\ = \; & u \end{aligned}$$

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Fiat-Shamir with Aborts (minor modifications)

Let R_q and A be public and $T : R_q^k \to \mathbb{Z}_q^{n_0}$ be a linear function, $n_0 \cdot \log_2(q) \approx 2\lambda$.

message

$\mathsf{sk} = s \leftarrow R^{k+\ell}$ small

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$c = H_c($ T (u) , , t) small

$z = s \cdot c + y$ (rejection sampling)

$\xrightarrow{\quad , = (\ u \ , z)\quad}$

compute $c = H_c($ T $(u), , t)$

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efficiency

$|T(u)| \approx 2\lambda = 256$
$|u| = nk \log_2 q \approx 40000$

$\xrightarrow{\quad , = (\ u\ \ , z)\quad}$

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security

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msg, sig $= (\ u\ , z)$

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functionality

$z = s \cdot c + y$ (rejection sampling)

, = (u , z)

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Naive Aggregation for FSwA Signature

Let R_q, A and $T : R_q^k \to \mathbb{Z}_q^{n_0}$ be public, H_c random oracle

message doc_1

$\mathsf{sk} = s_1, \mathsf{vk} = t_1$

$y_1, u_1, c_1 = H_c(T(u_1), \text{doc}_1, t_1)$

$z_1 = s_1 c_1 + y_1$ (rej. sampling)

$\sigma_1 = (u_1, z_1)$

message doc_2

$\mathsf{sk} = s_2, \mathsf{vk} = t_2$

$y_2, u_2, c_2 = H_c(T(u_2), \text{doc}_2, t_2)$

$z_2 = s_2 c_2 + y_2$ (rej. sampling)

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Naive idea: $\sigma = (u = u_1 + u_2, z = z_1 + z_2) \Rightarrow Az = t_1 c_1 + t_2 c_2 + u$

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$\text{pen}_1 = (u_1, z_1)$

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✖ Problem: How to compute c_1, c_2? Verifier doesn't know $T(u_1), T(u_2)$

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✖ Problem: How to compute c_1, c_2? Verifier doesn't know $T(u_1), T(u_2)$

⚙ Inter-active solution: agree on the same $u_1 = u_2$

⚙ Alternative: provide enough information by including all $T(u_j)$
(compression needed, see [DHSS20] and our scheme)

Outline

Linear Aggregation With Compression

Let R_q, A and $T : R_q^k \to \mathbb{Z}_q^{n_0}$ be public, H_c random oracle.

Given N signatures $\Sigma = (✎_j) = (u_j, z_j)_{j \in [N]}$ for verification keys $\mathsf{VK} = (t_j)_j$ on the messages $M = (🗎_j)_j$.

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$\mathsf{AggSig}(\mathsf{VK}, M, \Sigma)$: Compute $T(u_j) \quad \forall j \in [N]$
set $z = \sum_j z_j \in R_q^{\ell+k}$
and $u = \sum_j u_j \in R_q^k$
if $\|z\|_2$ small, return $✎ = (u, (T(u_j))_j, z)$;
else return $\perp$;

$\mathsf{AggVf}(\mathsf{VK}, M, ✎)$: Query $c_j = H_c(T(u_j), t_j, m_j) \quad \forall j \in [N]$
If $\|z\|_2$ small, and if $T(u) = \sum_j T(u_j)$
and if $A \cdot z = \sum_j (t_j \cdot c_j) + u$
return 1; else return 0;

Linear Aggregation With Compression

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and if $A \cdot z = \sum_j (t_j \cdot c_j) + u$
return 1; else return 0;

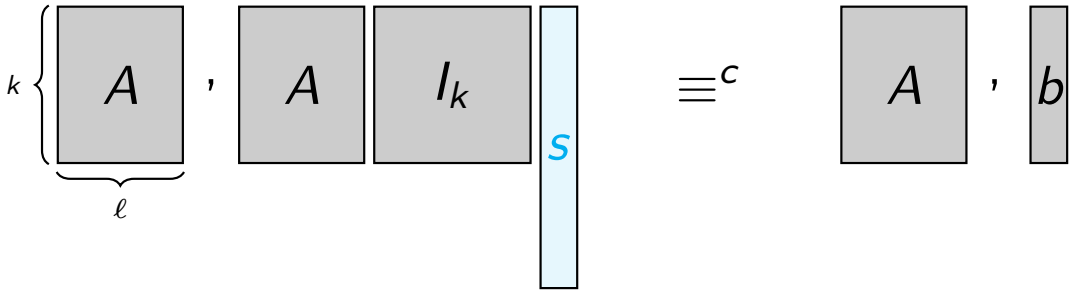
Correctness: Linearity of matrix-vector multiplication
Compactness: $|✒| \ll |\Sigma|$ $N = 10^3$ and Dilithium III: 43.7 KB vs. 2701 KB
Smallness: From rejection sampling $z_j \sim D_\alpha^{\ell+k}$, then $z \sim D_{\sqrt{N}\alpha}^{\ell+k}$

Overview

Hard Problems Over Module Lattices [LS15, BJRW20]

Let $R_q = \mathbb{Z}_q[x]/\langle x^n + 1\rangle$ and $k, \ell \in \mathbb{N}$

Module Learning With Errors (Module-LWE): Distinguish



where $s \in R^{\ell+k}$ is of small norm and $(A, b) \leftarrow U(R_q^{k\times\ell}) \times U(R_q^k)$.

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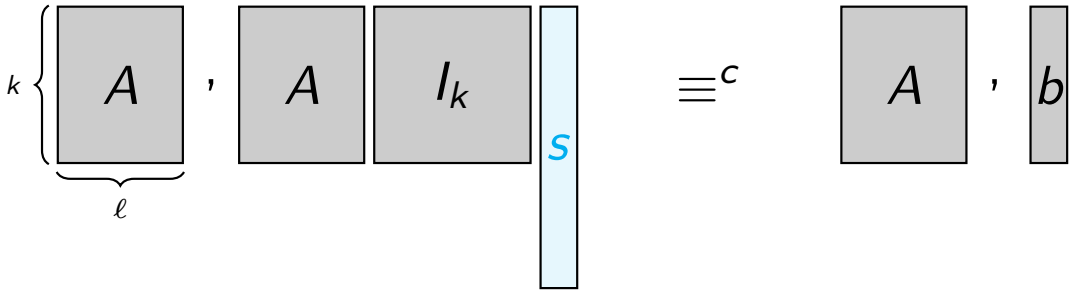
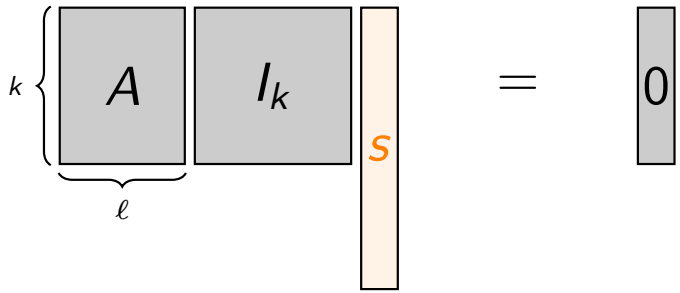


where $s \in R^{\ell+k}$ is of small norm and $(A, b) \leftarrow U(R_q^{k\times\ell}) \times U(R_q^k)$.

Module Short Integer Solution (Module-SIS): Given $A \leftarrow U(R_q^{k\times\ell})$, find s s.t.



where $s \in R_q^{\ell+k} \setminus \{0\}$ is of small norm.

Chosen-Key Model from [BGLS03]

$\Pi_{AS} = (\mathsf{KGen}, \mathsf{Sig}, \mathsf{Vf}, \mathsf{AggSig}, \mathsf{AggVf})$ aggregate signature scheme

$N \in \mathbb{N}$ number of signatures to aggregate

Challenger $\mathcal{B}$ | Adversary $\mathcal{A}$

$(\mathsf{sk}_N, \mathsf{vk}_N) \leftarrow \mathsf{KGen}$

$\mathsf{vk}_N \longrightarrow$

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Challenger $\mathcal{B}$ | Adversary $\mathcal{A}$

$(\mathsf{sk}_N, \mathsf{vk}_N) \leftarrow \mathsf{KGen}$

vk_N →

← (sign queries on vk_N)

→

Chosen-Key Model from [BGLS03]

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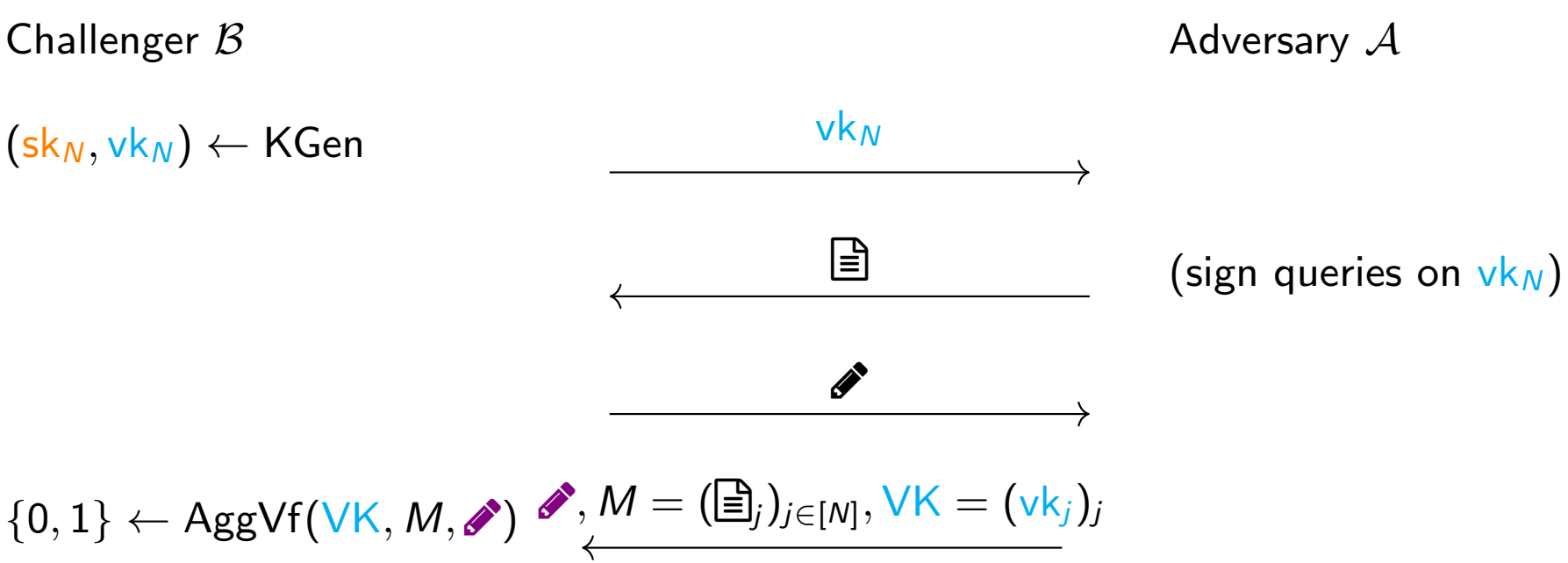

$\mathcal{A}$ wins the game if $1 \leftarrow \mathsf{AggVf}$ and m_N not queried before

Π_{AS} secure against existential forgery in chosen-key model if success proba of any PPT $\mathcal{A}$ is negligibly small

Security Proof: Statement

Random Oracle $H_c : \{0,1\}^* \to C$

Theorem (Security)

Assume the hardness of Module-LWE *and of* Module-SIS. *Then the aggregate signature Π_{AS} presented before is secure against existential forgery in the aggregate chosen-key model in the ROM. The advantage of some PPT adversary $\mathcal{A}$ against Π_{AS} is bounded above by*

$$\mathsf{AdvAggSig}_{\mathcal{A}} \leq \mathsf{Adv}_{\text{Module-LWE}} + N_q / |C| + \sqrt{N_q \cdot \mathsf{Adv}_{\text{Module-SIS}}} + \mathsf{negl}(n),$$

where $\mathcal{A}$ makes at most N_{H_c} queries to H_c and at most N_{Sig} queries to the signing oracle and $N_q = N_{H_c} + N_{\mathsf{Sig}}$.

Security Proof: Highlevel as in [DOTT20]

Game 0: Original security game:
challenger honestly generates $(\mathsf{sk}_N, \mathsf{vk}_N)$
and honestly answers signing queries

Game 1: Challenger simulates signing procedure
without using sk_N, only vk_N

Game 2 Challenger generates a lossy key $\mathsf{vk}_N \leftarrow U(R_q^k)$

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In Game 2: rewind challenger and adversary (apply General Forking Lemma) to obtain two different forgeries:

$$✎ = (u, (T(u_j))_j, z) \text{ and } ✎' = (u', (T(u_j)')_j, z'),$$

where $u = u'$, $T(u_j) = T(u_j)' \quad \forall j \in [N]$ and $c_j = c_j' \quad \forall j \in [N-1]$, but $c_N \neq c_N'$.

$$Az - t_N c_N = u + \sum_{j \in [N-1]} t_j c_j = u' + \sum_{j \in [N-1]} t_j c_j' = Az' - t_N c_N'$$

leads to solution to Module-SIS for matrix $[A|t_N]$.

Overview

Further work and open questions

Related work

- Inter-active aggregate signatures (aka multi-signatures)

Work in progress

- Realize relaxed aggregation: sequential aggregate signature
  aggregation follows a sequential order

Open questions **?**

- (almost) constant size and public aggregation
- Maybe tighter security proof using Abdalla et al. [AFLT16]
- Security proof in **quantum** random oracle model

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Thank you.


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